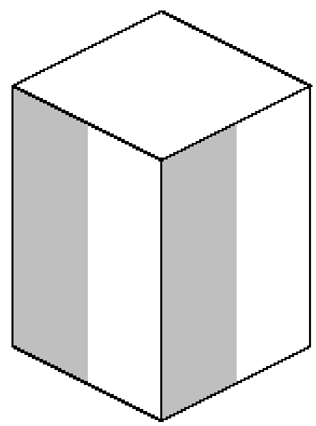
Exam Symmetry in Physics

Date	March 29, 2021
Time	9:00 - 11:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- All subquestions (a, b, etc) of the three exercises have equal weight
- Illegible answers will be not be graded
- Good luck!

Exercise 1

Consider the rectangular box with a square base depicted in the figure below:



The rectangular sides of the box all have the exact same pattern where the left half is grey and the right half is white.

(a) Identify all symmetry transformations (rotations and reflections) that leave this object invariant and call the group that they form G_{box}. Show that G_{box} is not isomorphic to the dihedral group D_4.

(b) Construct the character table of G_{box} and explain how the entries are obtained.

(c) Show whether G_{box} allows for an invariant vector, an invariant axial vector or neither. Explain how the answer can be understood from the figure and its symmetry transformations.

Exercise 2

Consider the symmetric group S_3 consisting of the permutations of three objects and view the three basis vectors of R^3 as the three objects that are permuted. This leads to the following three-dimensional rep D^L of S_3:

$$D^L(c) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad D^L(b) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Here the rep is only specified for the two generators c and b of S_3 which in cycle notation are given by $c = (123)$ and $b = (12)$.

(a) Show that D^L indeed forms a rep of S_3.

(b) Show using the characters of D^L that D^L is not an irrep of S_3 and decompose D^L into irreps of S_3 using the character table of S_3.

(c) Decompose $D^L \otimes D^L$ into irreps of S_3 using the character table of S_3.

(d) Identify the direction in R^3 that is invariant under the action of the rep D^L of S_3.

Exercise 3

Consider the group $SU(2)$ of unitary 2×2 matrices with determinant equal to 1. Consider its action on the spin states $|s, m_s\rangle$ through the operator

$$U(\theta, \hat{n}) = \exp\left(\frac{i}{\hbar}\theta\, \hat{n} \cdot \vec{S}\right),$$

where $\vec{S}$ denotes the spin operator.

(a) Write down the explicit matrix for S_z acting on the space of $|\frac{3}{2}, m_s\rangle$ states.

(b) Write down the explicit matrix for $U(\theta, \hat{n})$ acting on the space of $|\frac{3}{2}, m_s\rangle$ states for the specific case $\hat{n} = \hat{z}$, and determine the values of θ that describe all distinct $U(\theta, \hat{z})$.

(c) Use the character of the $s = \frac{3}{2}$ representation to show that it is equivalent, but not equal to its complex conjugate representation.